Faculty Critical Thinking Workshop #4
October 16, 2008
"Quantitative Reasoning, Scientific Reasoning, and Critical Thinking"

What is Quantitative Reasoning?

Quantitative reasoning/literacy is how one uses the mathematical knowledge one has to solve problems. Good quantitative reasoners recognize when quantitative or quantifiable concepts are presented to them, and recognize when the collection, analysis, and/or presentation of quantitative information would support a line of reasoning or bolster an argument. Quantitative reasoning is not the ability to perform formal manipulations on symbols, but rather the ability to be able to use such skills *in appropriate contexts*; this involves both recognizing traditional cues (natural language names for mathematical procedures and concepts) and understanding mathematical concepts well enough to apply them appropriately.

"Quantitative reasoning is the application of mathematical concepts and skills to solve real-world problems. The goal of Hollins' quantitative reasoning program focuses on students being able to use and read quantitative data, understand quantitative evidence, and apply basic quantitative skills in order to be effective members of society."

Hollins College Quantitative Reasoning Center
www1.hollins.edu/depts/qr/what.html

"Quantitative Literacy is not mathematics. It has almost nothing to do with mathematics as taught at the college level. It is about gaining the ability and habit to seek out quantitative information, to be able to analyze and critique it, and to use it in our public, personal, and professional lives. . . . QL recognizes the fact that quantitative information in the real world is messy, complex, often incomplete, sometimes overwhelming, and not infrequently appears to be contradictory. . . . There is still a lot to learn about how to choose and think about quantitative information when it is outside the idealized world of the mathematics classroom."

Macalester College Program in Quantitative Methods for Public Policy
www.macalester.edu/qm4pp/program/index.html/

"Some call it **Numeracy**, an expression first used in the UK's 1959 "Crowther Report" to include secondary school students' ability to reason and solve sophisticated quantitative problems, their basic understanding of the scientific method, and their ability to communicate at a substantial level about quantitative issues in everyday life. Others call it **Quantitative Literacy (QL)**, and describe this comfort, competency, and "habit of mind" in working with numerical data as being as important in today's highly quantitative society as reading and writing were in previous generations. Still others refer to it as **Quantitative Reasoning (QR)**, emphasizing the higher-order reasoning and critical thinking skills needed to understand and to create sophisticated arguments supported by quantitative data."

National Numeracy Network
serc.carleton.edu/nnn/resources/index.html

What is Involved in Teaching (Traditional) Mathematics?

a) Formal proofs and derivations (e.g., fundamental theorem of calculus)
b) Techniques for problem solving (e.g., factoring quadratic equations, equations in several variables)
c) Examples of application in different disciplines (e.g., physics, econometrics)

a) does not support the acquisition of quantitative reasoning skills. b) and c) do, but by themselves they are not sufficient. Acquisition of a skill is not sufficient for its systematic use in practical situations. ***Courses outside of mathematics need to emphasize the skill of applying appropriate mathematics skills.***

What Doesn't Work in Teaching Quantitative Literacy?

Stand-alone courses that are outside the regular curriculum, and interdisciplinary, team-taught courses that serve as a substitute for the regular curriculum.

Regarding the method of pairing a general, two-credit course in quantitative reasoning with a variety of disciplinary courses: "Preliminary results suggest that there is little evidence that the single session in quantitative reasoning each week had a significant impact on student ability to reason quantitatively."

> The problem apparently lay in articulating "a course that operates as a supplement to other existing courses . . . with these other courses that had very different goals." (Macalester eventually went to a model of stand-alone courses with an explicit emphasis on quantitative reasoning.) The same problem is found in writing across the curriculum programs. www.macalester.edu/qm4pp/program/index.html/

Dartmouth initiated a four-year Mathematics Across The Curriculum program in the late 1990s, which created 16 new, interdisciplinary courses to provide students extensive exposure to mathematical reasoning that went beyond the structure of traditional mathematics courses. They concluded that there was no significant different between participating students and their peers in: their subsequent total GPA, science GPA or engineering GPA, or in their overall continuation into a science, math or engineering major. What positive conclusions did they draw?

- They managed to made "mathematics more approachable and more interesting" for participating students and increased their interest in math.
- "In-class group work [helped] students to learn mathematics." It was necessary to develop assignments that were too complicated for one student to solve independently, but which were not too difficult to frustrate groups.
- Team-teaching provided faculty with "a valued opportunity for intellectual, pedagogical, and social exchange with their colleagues."
- "Professors in subsequent engineering courses rate[d] the teamwork abilities of [participating students] significantly above that of their conventionally prepared peers."
- Their strategy may have advantaged women.

In short, their costly model of team teaching may have helped students learn content, and it apparently served social ends, but it did not develop transferable skills other than building teamwork abilities.

Best Practices for Quantitative Literacy Programs

A two- (or sometimes three-) stage model has been imported at many schools from writing programs: the first course is taught by math professors and is introductory (sometimes students may be tested out; may be remedial), while the second is taught in the disciplines as part of a content course (Alverno, Bowdoin, Hollins, Wellesley, Wisconsin).

Sometimes these are telescoped into a single course taught by faculty from a range of departments on the WAC model (Augsburg, Carleton, Keene State, Lawrence, Northern Illinois, Skidmore, Trinity CT, UMass Boston). This runs the risk of looking more like a core course in mathematics for non-majors.

Carleton's writing program implemented a student portfolio project in 2001. The quantitative reasoning program piggybacks on this: faculty read a sample of papers every year to determine whether students choose to present quantitative arguments in cases where this would bolster their argument, and were able to analyze these arguments when they were presented.

> This is an assessment model, but it points to a way of operationalizing a goal of any quantitative reasoning program: how can students be prepared to make use of quantitative arguments when appropriate to their subject and goals?

Most initiatives to improve quantitative skills extend beyond quantitative reasoning as we understand it. They include, for example, improving students abilities in the formal manipulation of symbols, teaching higher-order and complex mathematics, diversifying and increasing enrollments in math-intensive fields – these are quantitative literacy programs rather than ***quantitative reasoning*** ones. Consider this list of way to "best prepare quantitatively literate students":

1. Place concepts in context
2. Use multiple representations
3. Work in groups
4. Use appropriate technology
5. Do in-depth problems that last more than one day

serc.carleton.edu/quantskills/methods/quantlit/index.html

Or this list of characteristics of the core course in quantitative reasoning course at De Paul:

A) Multidisciplinary contexts
B) Integration of technology
C) Active learning strategies
D) Faculty sharing
E) Emphasis on reasoning

It should now be easy to distinguish strictly pedagogical recommendations from broader, curricular recommendations. Both of these lists are highly relevant to our purposes (other, similar lists produced elsewhere are less so). Notice similarities to best practices in teaching writing and critical thinking (1, 3, 5, A, C, E). Occasionally writing is taught in "paired" courses (D), and sometimes technology (4, D) literacy is added to writing courses. The ability to use multiple representations (2) is shared with other symbol-intensive fields (e.g., phonology) and is an important skill for critical thinking in certain contexts (e.g., use of charts and graphs), but often receives insufficient attention.

Wellesley College's "Quantitative Reasoning Overlay" requirement (modeled on the writing across the curriculum model) is developed around 5 goals. Consider two:

1) Authenticity. Students should have experience in using authentic numerical data. The experience should arise naturally in the context of the course and actually advance the work of the course.
2) Understanding. A student's experience with data analysis should not be limited to rote application of some involved statistical procedure. Rather, students should understand enough of what they are doing so that their experience of data analysis is likely to stay with them, at least as a residue of judgment and willingness to enter into similar data analysis in the future.

The understanding goal demonstrates how the QR program contributes to critical thinking instruction: it takes students beyond rote manipulation and establishes a foundation for continued exercise of the skills learned in every QR course.

Attached Examples

Consider two examples:

- Excerpt from a syllabus for an introductory course in quantitative reasoning (p. 7). A stand-alone course isn't necessary: this gives examples of basic skills that could be taught (addressed) as necessary in individual courses, or a curricular decision could be made to address them more systematically in a core curriculum or a sequence of courses in a major.
- An introduction to correlation (p. 8) that eschews underlying math and focuses on practical use. When teaching for quantitative literacy, practical rules of thumb are more important than derivations and proofs that occupy so much time in mathematics and other quantitative courses.
- Sampon-Nicolas' (unfortunately rare) example of a good quantitative reasoning module in a humanities course (pp. 9-10).
- A more problematic example comes from Bohland's course in political economy (pp. 11-13).
- Seattle University capstone course problem (p. 14) gives an excellent example of assessing the skills that students need to acquire in their college education, and the ways they may fall short.

Questions

1. Are there key quantitative concepts and techniques that all students should know as preparation for life in an information-rich society? If so, what are they?
2. How (and where) are these currently taught in the Bethel curriculum?
    - Is the basic skills approach analogous to the great books approach to culture, or the normative grammar approach to writing?
3. Alternately: can discrete skills be taught in the disciplines on an as-needed basis?
    - If quantitative reasoning is the goal, should our emphasis be on having students learn how to use new techniques to support arguments, instead of or in addition to providing them with a box of basic tools that they can apply as the context warrants?

Scientific Reasoning

Like quantitative reasoning, this can be taught in a stand-alone course for non-majors, or in one for majors. Textbooks designed for stand-alone courses include:

- Giere, Ronald, John Bickle, and Robert Mauldin, *Understanding Scientific Reasoning* (5th edition), Thomson, 2006.
- Holowchak, Andrew, *Critical Reasoning and Science: Looking at Science with an Investigative Eye*, University Press of America, 2007.

Both are heavily inspired by philosophy of science curricula, but extend beyond this by offering brief introductions to statistics, causation, the history of science, some discussion of pseudo-science, and the "scientific method", i.e., they provide models for understanding how scientific research is conducted.

See excepts at the end of this handout from both books (pp. 14-15): both provide a meta-model of scientific models; their meta-models are similar. In these excerpts, Holowchak provides students with a workbook-type handout (rather like I often do in these workshops), and Giere et al. provides a nice flowchart: both help to guide students' reasoning about scientific models. Similar strategies can be used to model key elements of reasoning in any discipline.

Every discipline has methods of reasoning that are particular (perhaps unique) to that discipline. **Critical thinking includes discipline-specific as well as general components.** Over the next year, you will be asked to develop critical tinking tests for your majors – a good way to begin the process is to think about way to explicitly target (and assess) discipline-specific modes of reasoning in your own classes.

Easy-To-Borrow Exercises or Tips

I'm not aware of textbooks designed for use as supplemental texts in content courses in the sciences, or which give model exercises that can be used in class for, e.g., 10 minutes or a single class period. But as with teaching argumentation, available textbooks sometimes will provide models or inspiration for particular exercises.

Consider **Robert Ableson, *Statistics as Principled Argument***, 1995: this could be helpful for connection between the use of mathematics and argumentation; for helping students get beyond calculation and think about its use (quantitative reasoning); as an auxiliary textbook; because it may help with CT test scores. It's not a book that you can skim for 20 minutes and come out with an exercise to use in your class: it really needs to be read cover-to-cover and digested.

A Comparison

Note that science isn't a skill or a tool – it's a (set of) discipline(s), for which particular skills are important. So teaching scientific reasoning really entails targeting and teaching skills rather than teaching scientific content. In this respect it is like quantitative reasoning: courses in the latter often combine goals related to the teaching of mathematics with quantitative reasoning goals.

A comparison of the contents of three textbooks (Giere et al, Holowchak, and Aufmann et al., *Mathematical Thinking and Quantitative Reasoning*) will illustrate some broad patterns in textbooks:

Giere et al.	**Holowchak**	**Aufmann et al.**
1) Why understand scientific reasoning?	1) Science and critical reasoning	1) Problem solving
2) Understanding and evaluating theoretical hypotheses	2) Science and truth	2) Logic and its applications
3) Historical episodes	3) Causation	3) Algebraic models
4) Marginal science	4) Science and progress	4) Measurement and geometric models
5) Statistics and probability	5) Scientific models	5) Linear models
6) Evaluating statistical hypotheses	6) Statistical hypotheses	6) Non-linear models
7) Causal models	7) Causal hypotheses	7) Mathematics of finance
8) Evaluating causal hypotheses	8) Science and Non-Science	8) Probability and statistics
9) Models of decision making		9) Apportionment and voting
10) Evaluating Decisions		10) Mathematics of graphs

Note the overlap with the MAPP and CAT tests, which cover: causal explanations, investigating questions of causation, interpreting graphs, and understanding correlational data.

Quantitative Reasoning and Scientific Reasoning: Conclusion

It appears that these overlap (Giere et al. and Holowchak devote significant space to strictly mathematical skills and their use, application, or interpretation), and in addition scientific reasoning requires knowledge of discipline-specific modes of thinking.

Reasoning in non-quantitative fields overlaps some of the topics associated above with quantitative fields (causation, correlation, decision-making, evaluation and comparison of models, truth). If courses in every discipline, especially in the common core, focus some explicit attention on discipline-specific modes of reasoning, students will see the same methods in quite diverse fields. This should help them to transfer skills across domains.

Summary

How does one teach so as to improve quantitative and scientific reasoning?

- Keep track of the goals of any given exercise or assignment. What skills does it target?
- Use teaching methods that support critical thinking (active learning; interesting, real-life examples; writing-to-learn; collaboration; etc.).
- Focus on application, not just on formal methods of calculation.
- Ensure that students understand reasons for and constraints on methods. (Seattle Univ. example)
- From a curricular perspective: make sure students encounter a good mix of well- and ill-structured problems within and across courses.

From a syllabus by Curt White (De Paul University)

Week 1: Introduction to mathematical modeling and linear models in particular

Mathematical modeling is a collection of mathematical techniques to facilitate predictions for planning or simulation purposes. The basic approach is to take existing data and fit a mathematical object (an equation, graph, shape, or algorithm) to the data. If the data fits reasonably well, one can make predictions in between existing data points (interpolations) or beyond the existing data (extrapolations). Confidence in predictions is tempered by the goodness of fit of the model and by how one is away from existing data. Generally, the farther one is from existing data, the less confidence one has in predictions. This week introduces the general concept of modeling, reviews linear functions, and covers linear models. Excel is used to add trendlines, get their equations, and use them to make predictions. You are introduced to the R-squared value as a measure of goodness of fit.

Week 2: Making and interpreting graphs

Graphs are an extremely powerful and useful way of presenting and analyzing quantitative information. The week's focus is on learning how to make three types of graphs in Excel: pie charts, bar charts (including multiple bar charts), and XY-graphs. You learn when each type of graph is appropriate, how to describe and interpret each type of graph, and how to critique misleading graphs from the media. The important vocabulary for describing XY graphs is introduced: increasing, decreasing, constant absolute maximum, absolute minimum, relative maximum, relative minimum, periodicity, increasing at an increasing or decreasing rate, and decreasing at an increasing or decreasing rate.

Week 3: Percentages

The major topic of the week is the use of percentages in quantitative work. You are expected to master five types of percentage problems that arise in everyday quantitative work: percentage-as-part-of-a-whole problems, percent change, "percent more than" problems, successive percent change problems, and reverse percentage change problems.

Week 5: Absolute and relative quantities

Absolute quantities refer generally to raw numbers and counts; relative quantities are usually ratios of absolute quantities that normalize absolute quantities for comparison. Examples of relative quantities are rates and percentages. The first week focuses on the distinction between these two kinds of quantities, especially the importance of relative quantities. Along the way, Excel is introduced. You learn how to use cell reference, how to sort, and how to fill. You will also learn how to make maps with a geographic mapping tool.

Week 6: Localized trendlines

We will revisit trendlines and specifically learn how to identify localized trends. As part of this topic, we will also learn how to animate graphs in PowerPoint.

ISP 121

Introduction to Correlation

From a lecture by Curt White, DePaul University

Correlation

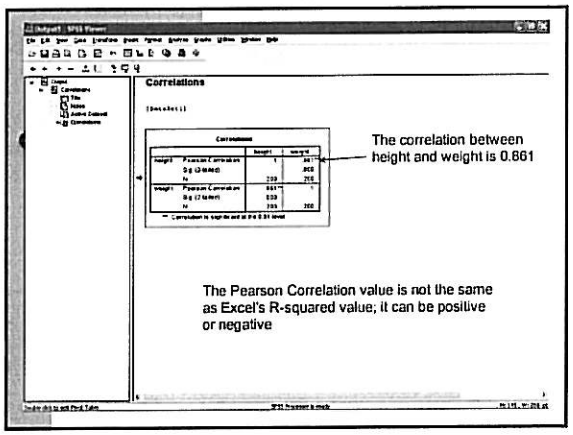
- The news is filled with examples of correlation
  - If you eat so many helpings of tomatoes…
  - One alcoholic beverage a day…
  - Driving faster than the speed limit…
  - Women who smoke during pregnancy…
  - If you eat only fast food for 30 days…
  - If your parents did not have offspring…

How Do You Calculate Correlation in Excel?

- Make an XY scatterplot of the data, putting one variable on the x-axis and one variable on the y-axis.
- Insert a linear trendline on the graph and include the R^2 value
- Interpret the results

Interpreting the Results

- The higher the R^2 value, the better
- If you only have a few data points, then you need a higher R^2 value in order to conclude there is a correlation
- Crude estimate: $R^2 > 0.5$, most people say there is a correlation; $R^2 < 0.3$, the correlation is essentially non-existent
- R^2 between 0.3 and 0.5?? Gray area!



What Can We Conclude?

- If two variables are correlated, then we can *predict* one based on the other
- But correlation does **NOT** imply cause!
- It might be the case that having more education causes a person to earn a higher income. It might be the case that having higher income allows a person to go to school more. There could also be a third variable. Or a fourth. Or a fifth…

Hollins College

French 356 French for International Business
Professor Annette Sampon-Nicolas

Applied Quantitative Reasoning Module 1
Analysis of Statistical Data of Exports

French for International Business studies business French with an emphasis on international trade. Topics discussed include globalization, developing countries, fair trade, corporate social and environmental responsibility, and ethical investments. Projects include a researched debate on globalization, studies of socially responsible companies in the US, Europe and of French CAC 40 (Paris Stock exchange) companies. Final project is the creation of a socially responsible company. Students present their product or service, their marketing campaign, their mission, their human resource policies, as well as their social and environmental activities in the community.

Goal of module I for French 356: To learn together how to research the economies of developing countries in order to prepare a debate on globalization from an informed point of view.

Goal of class applied quantitative reasoning project: To read, study and analyze textual and statistical data regarding **exports** and their role in the economy of Vietnam over the past 15 years and to represent these changes in charts, table and written form.

Case study: Vietnam: Integration with the global economy: We will work together on the study of Vietnam to develop research and analytical skills.

Directions:

1. We will be studying Vietnam as a class. Our first assignment will be to read a prose document about the economic and financial situation of Vietnam in 2003 and transform the information gleaned from the text into chart and table form. Go to http://www.dree.org/ Go to *Outside markets* and click on *Information by countries/region*. Click on *Asia/Pacific*. Then click on *Vietnam*. Go to *Country Information*. Scroll down to *Economic and Financial Situation* and click on *Fiches de Synthèse* and go to the economic and financial situation of Vietnam 2003. Print out the text, read it, and bring it to class where we will discuss the information.

2. Data collection, analysis, and graphing are essential in many fields and are useful tools. Before we are able to discuss the materials, we will first go over French vocabulary used in articles on international trade and in statistical analysis. As one of our goals is the transforming of information we have read into charts, we will review and discuss different forms of charts and graphs to see which types will be most useful and why. We will also review how to calculate percentage changes over time in order to see the extent of changes. We will do exercises on these areas in class with partners and as a group. You will explain which graph

you think explains your data best and why. Tables, charts and graphs are convenient ways to clearly show your data. Be sure to consider how to best show your results with appropriate graph forms. Be sure to give your charts and graphs an appropriate title that explains what the data measures. On line and bar graphs, the x and y axes must be appropriately labeled with correct unit of measure. The easiest way to create a graph is to enter your data into a spreadsheet program such as Excel. This program will generate graphs from the data you enter. We will be using the computer lab at least once early in the semester.

3. Your next assignment is to find out everything you can about the Vietnamese economy over the past 15 years. Identify possible sources and location of information (print, non-print, surveys, on-line, books) Look for documents, charts, and articles on trade and development. Bring your list of sources to class to share.

4. Acquire information pertinent to the study of Vietnam through your research. Identify relationships among variables within charts, graphs and tables. What kinds of information related to trade have you found? What kind of information do the charts or tables provide? Find patterns in the data. What can you learn about changes over time? If possible, compare information about a topic drawn from two or more sources to see if it is identical, similar, parallel, or inconsistent, unrelated or contradictory.

5. Research exports over past 15 years. Which products are exported? How are exports divided up? Agrifoodstuffs, fuel and energy, manufacturing, mineral products, chemical and plastic products, wood and paper, textiles, clothes, base metals, machinery equipment, technology, and other goods? Have exports increased or decreased? By how much? Which products have increased? Design a graph to show changes in exports over the past 15 years. Who are Vietnam's trade partners? Have they changed over the years? How do you explain these changes? Analyze the exchanges between Vietnam and it partners.

6. Look at the changes in exports over the past 15 years. What might explain these changes? Can you make any conclusion? Find the GDP of the country over the past 15 years. How has it changed? Draw conclusions about economic growth within the economy of Vietnam. What has been the general economic situation of Vietnam over the past 15 years? Can you make any forecasts about the future?

7. Your final assignment will be to write a five page paper incorporating the information you have analyzed. Do not forget to cite your sources. Your paper needs to include a statement of the problem or question your were researching, an explanation of the methods you used, and a summary in both prose and chart forms, of the results of your research and study. Do not forget to also address questions of any limitations encountered and possible improvements we might make to the procedures and or results of your research project.

From: Global Systems/International Political Economy
Course at Hollins College (Dr. Jon Bohland)

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Quantitative Take Home One: INTL 220 NAME_____

From Chart One

1. Based on the above data, list three states that fall into the category of Under $1 a Day in terms of GNP per capita and discuss any geographic trends you note from the data.

(Since I haven't given you the colored version, I'll give you the list of countries:
Mali, Niger, Chad, Central Afr. Rep., Nigeria, Sierra Leone, Guinea-Bissau, Banjul, Togo, Burkina Faso
Yemen, Eritrea, Ethiopia, Kenya, Uganda, Tanzania, Malawi, Mozambique, Madagascar
Bangladesh, Tajikistan, Vietnam, Cambodia
-- Carl)

2. Researching on your own, what is the mathematical formula and data set used to determine GNP per capita for each state?

From Chart Two

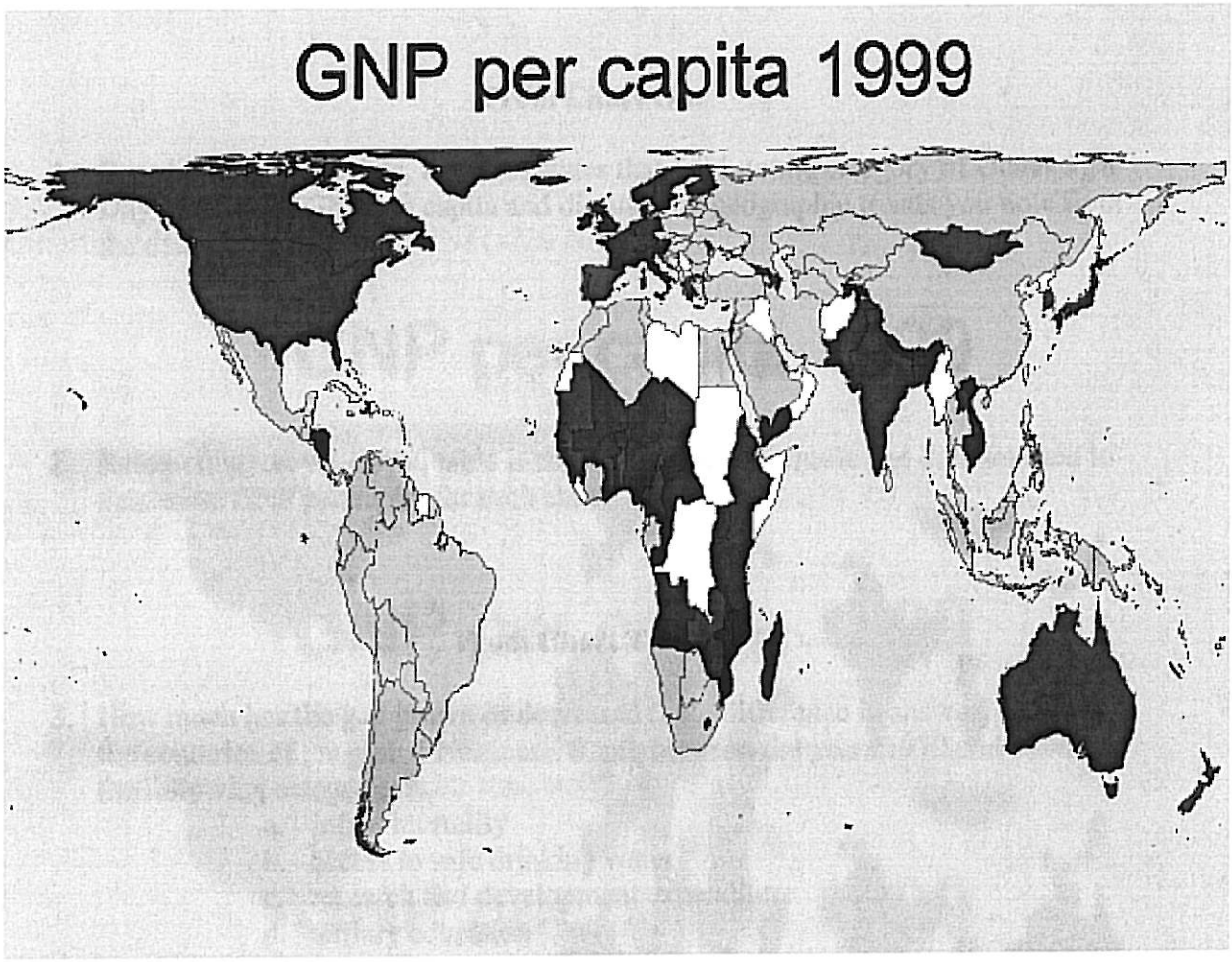
3. How much has the gap grown or decreased (note difference in answer) between the countries of the global North and South between the years 1970 and 1990 in the following categories?
   a. infant mortality
   b. access to safe drinking water
   c. research and development expenditure
   d. tertiary education

a._____ b. _____ c. _____ d._____

4. Comparing the human progress chart and the human survival chart, what broad trends do you notice in regards to the twenty years of data tabulated?

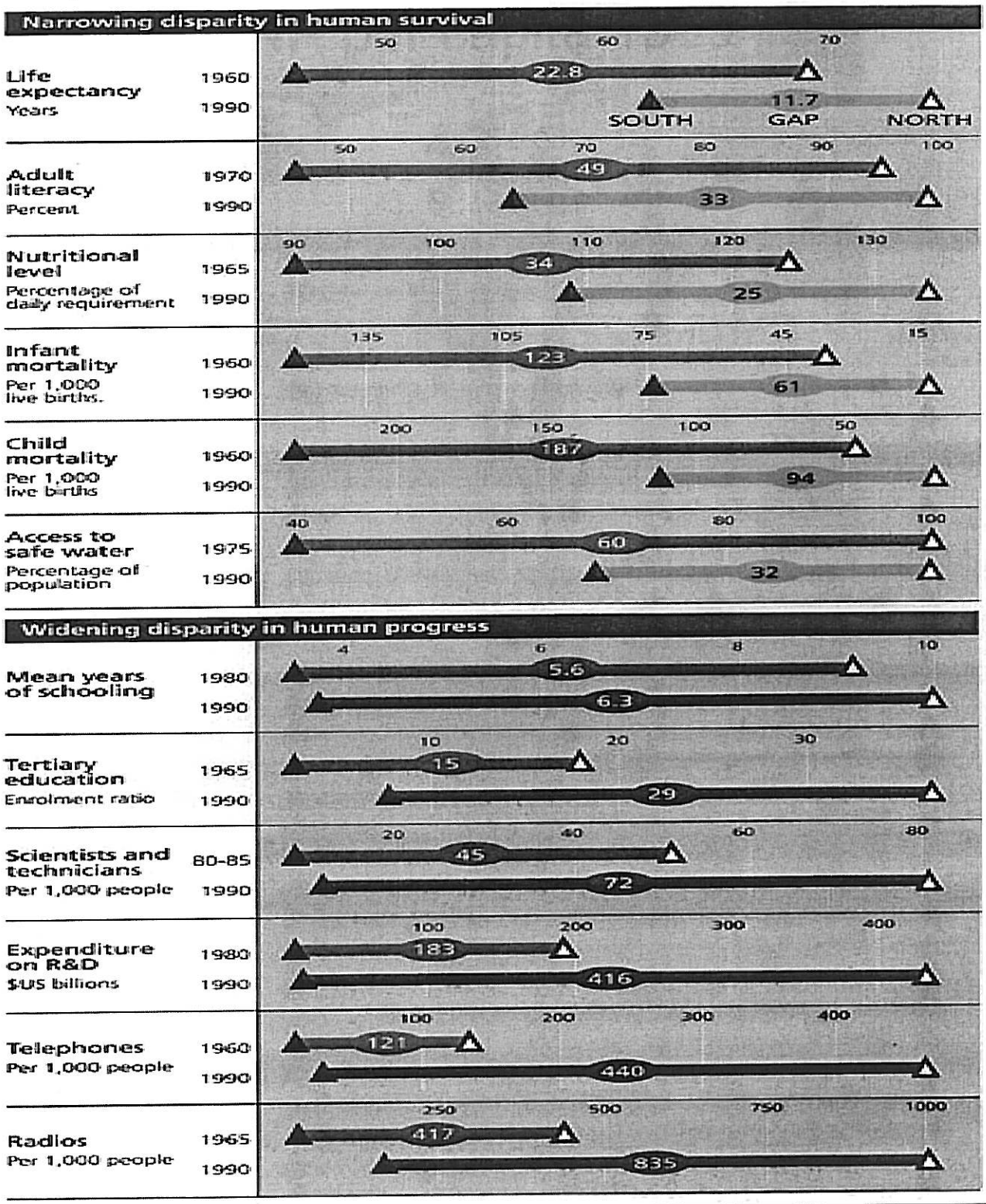
5. Why do you suppose the above trend relates both to the changes in the world economy and is important to consider in relation to issues of inequality in the world?

CHART ONE



GNP per cap
- Under $1 per day
- Under $2 per day
- 730 - 5000
- 5000 - 10000
- 10000 - 20000
- 20000 +
- No Data

FIGURE 3.5
North-South gaps in human development
Change in absolute disparity



(14)

ASSESSING CRITICAL THINKING: AN ASSESSMENT PROJECT IN FINANCE AT SEATTLE UNIVERSITY

THE ASSESSMENT TASK

Senior finance majors in a capstone course were given a case problem in which they played the role of financial advisor to husband and wife clients faced with an investment dilemma. The retiring couple were considering two choices for investing the wife's lump sum payment ($155,673.53) from a 401 K plan.

- *Choice A*: Buy an "Immediate Single-Life Annuity" which promises a monthly payment of $1225.85 for the rest of her life and a lump sum payout to a designated survivor of $37,000.
- *Choice B*: Invest the lump sum in a growth mutual fund which was yielding 10 percent annual return at the time (the case study was developed in 2000). At 10 percent, the plan would pay the same $1225.85 but return the original principle at time of the wife's death.

Students were given the following task: *You have crunched the numbers on the two plans, analyzed the results, and begun to formulate some conclusions and advice. Write a 2-3 page memo to David and Marilyn in which you analyze the benefits and risks of each plan and offer advice on how they can make a decision. Explain to David and Marilyn the different methods of analysis you used, why you used them, and what useful information each method revealed. Attach to your memo any visuals or graphics that would be useful to them in comparing the two plans. Note that David and Marilyn are well-educated college graduates but they have no background in finance.*

THE METHOD

Members of the finance faculty met for an all-day meeting to create a primary trait scoring rubric for the assignment and staff-grade the memos. Following the grading, the department held a detailed discussion of their findings.

THE FINDINGS

- Approximately half of the students scored in a range which the faculty consider cause for concern while even top-half students showed considerable critical thinking weaknesses
- Almost all students used tools and methodologies covered in the finance curriculum (NPV analysis, calculating an IRR, etc.) but many students used them randomly, often applying them to extraneous data, and revealing no purpose or goal in the calculation
- Many students failed to address the client's problem and provide the requested financial counsel
- Many student were unable to translate finance concepts/methods into lay language
- Generally students failed to construct rhetorically useful graphics

FEEDBACK LOOP TO IMPROVE CURRICULUM AND INSTRUCTION

- Finance faculty realized that instructors of 300-level finance courses preceding the capstone course assigned primarily algorithmic homework sets based on formulas and quantitative story problems (well-structured problems)
- Faculty are developing methods to place more ill-structured problems into these earlier courses through writing assignments or small group activities requiring group speakers to present extended arguments in support of solutions.
- Overall goal is more balanced attention to mastery of algorithmic tools and "big-picture" use of tool